5-1 Trigonometric Identities

Find the value of each expression using the given information.

1. If cot $\theta = \frac{5}{7}$, find tan θ.

SOLUTION:

$$\tan\theta = \frac{1}{\cot\theta} = \frac{1}{\frac{5}{7}} = \frac{7}{5}$$

3. If tan $\alpha = \frac{1}{5}$, find cot α.

SOLUTION:

$$\cot\alpha = \frac{1}{\tan\alpha} = \frac{1}{\frac{1}{5}} = \frac{5}{1} = 5$$

5. If cos $x = \frac{1}{6}$ and sin $x = \frac{\sqrt{35}}{6}$, find cot x.

SOLUTION:

$$\cot x = \frac{\cos x}{\sin x} = \frac{\frac{1}{6}}{\frac{\sqrt{35}}{6}} = \frac{1}{\sqrt{35}} = \frac{1}{\sqrt{35}} \cdot \frac{\sqrt{35}}{\sqrt{35}} = \frac{\sqrt{35}}{35}$$

7. If $\csc \alpha = \frac{7}{3}$ and $\cot \alpha = \frac{2\sqrt{10}}{3}$, find $\sec \alpha$.

SOLUTION:

Use the reciprocal identity $\csc\alpha = \frac{1}{\sin\alpha}$ to find $\sin \alpha$.

$$\begin{aligned}\csc\alpha &= \frac{1}{\sin\alpha}\\ \frac{7}{3} &= \frac{1}{\sin\alpha}\\ 7\sin\alpha &= 3\\ \sin\alpha &= \frac{3}{7}\end{aligned}$$

Use the quotient identity $\cot\alpha = \frac{\cos\alpha}{\sin\alpha}$ to find $\cos \alpha$.

$$\begin{aligned}\cot\alpha &= \frac{\cos\alpha}{\sin\alpha}\\ \frac{2\sqrt{10}}{3} &= \frac{\cos\alpha}{\frac{3}{7}}\\ \frac{2\sqrt{10}}{3}\cdot\frac{3}{7} &= \cos\alpha\\ \frac{2\sqrt{10}}{7} &= \cos\alpha\\ \cos\alpha &= \frac{2\sqrt{10}}{7}\end{aligned}$$

Use the reciprocal identity $\sec\alpha = \frac{1}{\cos\alpha}$ to find $\sec \alpha$.

$$\begin{aligned}\sec\alpha &= \frac{1}{\cos\alpha}\\ &= \frac{1}{\frac{2\sqrt{10}}{7}}\\ &= \frac{7}{2\sqrt{10}}\\ &= \frac{7}{2\sqrt{10}}\cdot\frac{\sqrt{10}}{\sqrt{10}}\\ &= \frac{7\sqrt{10}}{2(10)}\\ &= \frac{7\sqrt{10}}{20}\end{aligned}$$

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Find the value of each expression using the given information.

9. sec θ and cos θ; tan θ = –5, cos θ > 0

SOLUTION:

Use the Pythagorean Identity that involves tan θ to find sec θ.

$$\begin{aligned}\tan^2\theta + 1 &= \sec^2\theta \\ (-5)^2 + 1 &= \sec^2\theta \\ 26 &= \sec^2\theta \\ \pm\sqrt{26} &= \sec\theta \\ \sec\theta &= \pm\sqrt{26}\end{aligned}$$

Since we are given that cos θ is positive, the reciprocal function sec θ must be positive, so $\sec\theta = \sqrt{26}$.

Use the reciprocal identity $\sec\theta = \frac{1}{\cos\theta}$ to find cos θ.

$$\begin{aligned}\sec\theta &= \frac{1}{\cos\theta} \\ \sqrt{26} &= \frac{1}{\cos\theta} \\ \cos\theta &= \frac{1}{\sqrt{26}} \\ \cos\theta &= \frac{\sqrt{26}}{26}\end{aligned}$$

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11. tan θ and sin θ; sec θ = 4, sin θ > 0

SOLUTION:

Use the reciprocal identity $\sec\theta = \frac{1}{\cos\theta}$ to find cos θ.

$$\sec\theta = \frac{1}{\cos\theta}$$
$$4 = \frac{1}{\cos\theta}$$
$$\cos\theta = \frac{1}{4}$$

Use the Pythagorean Identity that involves sec θ to find tan θ.

$$\tan^2\theta + 1 = \sec^2\theta$$
$$\tan^2\theta + 1 = (4)^2$$
$$\tan^2\theta = (4)^2 - 1$$
$$\tan^2\theta = 15$$
$$\tan\theta = \pm\sqrt{15}$$

Since sin θ is positive, and $\cos\theta = \frac{1}{4}$ is positive, tan θ must be positive. Therefore, $\tan\theta = \sqrt{15}$

Use the quotient identity $\tan\theta = \frac{\sin\theta}{\cos\theta}$ to find sin θ.

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$
$$\sqrt{15} = \frac{\sin\theta}{\frac{1}{4}}$$
$$\frac{\sqrt{15}}{4} = \sin\theta$$
$$\sin\theta = \frac{\sqrt{15}}{4}$$

13. cos θ and tan θ ; csc θ = $\frac{8}{3}$, tan θ > 0

SOLUTION:

Use the Pythagorean Identity that involves csc θ to find cot θ.

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$$\cot^2\theta + 1 = \csc^2\theta$$
$$\cot^2\theta + 1 = \left(\frac{8}{3}\right)^2$$
$$\cot^2\theta = \frac{64}{9} - 1$$
$$\cot^2\theta = \frac{55}{9}$$
$$\cot\theta = \pm\sqrt{\frac{55}{9}}$$
$$\cot\theta = \pm\frac{\sqrt{55}}{3}$$

Since we are given that tan θ is positive, cot θ must also be positive. Therefore, $\cot\theta = \frac{\sqrt{55}}{3}$.

Use the reciprocal identity $\tan\theta = \frac{1}{\cot\theta}$ to find tan θ.

$$\tan\theta = \frac{1}{\cot\theta} = \frac{1}{\frac{\sqrt{55}}{3}} = \frac{3}{\sqrt{55}} = \frac{3}{\sqrt{55}} \cdot \frac{\sqrt{55}}{\sqrt{55}} = \frac{3\sqrt{55}}{55}$$
.

Use the reciprocal identity $\csc\theta = \frac{1}{\sin\theta}$ to find sin θ.

$$\csc\theta = \frac{1}{\sin\theta}$$
$$\frac{8}{3} = \frac{1}{\sin\theta}$$
$$\sin\theta = \frac{1}{\frac{8}{3}}$$
$$\sin\theta = \frac{3}{8}$$

Use the quotient identity $\cot\theta = \frac{\cos\theta}{\sin\theta}$ to find cos θ.

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$
$$\frac{\sqrt{55}}{3} = \frac{\cos\theta}{\frac{3}{8}}$$
$$\frac{\sqrt{55}}{8} = \cos\theta$$
$$\cos\theta = \frac{\sqrt{55}}{8}$$

15. cot θ and sin θ; sec $\theta = -\frac{9}{2}$, sin $\theta > 0$

SOLUTION:

Use the Pythagorean Identity that involves sec θ to find tan θ.

$$\tan^2\theta + 1 = \sec^2\theta$$
$$\tan^2\theta + 1 = \left(-\frac{9}{2}\right)^2$$
$$\tan^2\theta = \frac{81}{4} - 1$$
$$\tan^2\theta = \frac{77}{4}$$
$$\tan\theta = \pm\sqrt{\frac{77}{4}}$$
$$\tan\theta = \pm\frac{\sqrt{77}}{2}$$

Since we are given that sec θ is negative, we know that cos θ is negative. We are also given that sin θ is positive. Since cos θ is negative and sin θ is positive, tan θ is negative. Therefore, $\tan\theta = -\frac{\sqrt{77}}{2}$.

Use the reciprocal identity $\cot\theta = \frac{1}{\tan\theta}$ to find cot θ.

$$\begin{aligned}\cot\theta &= \frac{1}{\tan\theta}\\ &= \frac{1}{-\frac{\sqrt{77}}{2}}\\ &= -\frac{2}{\sqrt{77}}\\ &= -\frac{2}{\sqrt{77}}\cdot\frac{\sqrt{77}}{\sqrt{77}}\\ &= -\frac{2\sqrt{77}}{77}\end{aligned}$$

Use the reciprocal identity $\sec\theta = \frac{1}{\cos\theta}$ to find cos θ.

$$\sec\theta = \frac{1}{\cos\theta}$$
$$-\frac{9}{2} = \frac{1}{\cos\theta}$$
$$\cos\theta = -\frac{2}{9}$$

Then use the Pythagorean Identity that involves cos θ to find sin θ.

$$\cos^2\theta + \sin^2\theta = 1$$
$$\left(-\frac{2}{9}\right)^2 + \sin^2\theta = 1$$
$$\sin^2\theta = 1 - \frac{4}{81}$$
$$\sin^2\theta = \frac{77}{81}$$
$$\sin\theta = \pm\sqrt{\frac{77}{81}}$$
$$\sin\theta = \pm\frac{\sqrt{77}}{9}$$

Since we are given that sin θ is positive, sin θ = $\frac{\sqrt{77}}{9}$.

Find the value of each expression using the given information.

17. If csc θ = –1.24, find $\sec\left(\theta - \frac{\pi}{2}\right)$.

SOLUTION:

$$\begin{aligned}\sec\left(\theta-\frac{\pi}{2}\right) &= \sec\left[-\left(\frac{\pi}{2}-\theta\right)\right] \\ &= \sec\left(\frac{\pi}{2}-\theta\right) && \text{Odd-Even Identity} \\ &= \csc\theta && \text{Cofunction Identity} \\ &= -1.24 && \csc\theta = -1.24\end{aligned}$$

19. If tan θ = –1.52, find $\cot\left(\theta - \frac{\pi}{2}\right)$.

SOLUTION:

$$\begin{aligned}\cot\left(\theta-\frac{\pi}{2}\right) &= \cot\left[-\left(\frac{\pi}{2}-\theta\right)\right] \\ &= -\cot\left(\frac{\pi}{2}-\theta\right) && \text{Odd-Even Identity} \\ &= -\tan\theta && \text{Cofunction Identity} \\ &= -(-1.52) && \tan\theta = -1.52 \\ &= 1.52\end{aligned}$$

21. If cot x = 1.35, find $\tan\left(x-\frac{\pi}{2}\right)$.

SOLUTION:

$$\begin{aligned}\tan\left(x-\frac{\pi}{2}\right) &= \tan\left[-\left(\frac{\pi}{2}-x\right)\right] & \\ &= -\tan\left(\frac{\pi}{2}-x\right) & \text{Odd-Even Identity} \\ &= -\cot x & \text{Cofunction Identity} \\ &= -1.35 & \cot x = 1.35\end{aligned}$$