5-1 Trigonometric Identities

Simplify each expression.

23. $\csc x - \cos x \cot x$

SOLUTION:

$$\begin{aligned}\csc x - \cos x \cot x &= \frac{1}{\sin x} - \cos x \frac{\cos x}{\sin x} \\ &= \frac{1}{\sin x} - \frac{\cos^2 x}{\sin x} \\ &= \frac{1 - \cos^2 x}{\sin x} \\ &= \frac{\sin^2 x}{\sin x} \\ &= \sin x\end{aligned}$$

25. $\dfrac{\tan x + \sin x \sec x}{\csc x \tan x}$

SOLUTION:

$$\begin{aligned}\frac{\tan x + \sin x \sec x}{\csc x \tan x} &= \frac{\tan x + \sin x \cdot \frac{1}{\cos x}}{\csc x \cdot \tan x} \\ &= \frac{\tan x + \frac{\sin x}{\cos x}}{\csc x \cdot \tan x} \\ &= \frac{\tan x + \tan x}{\csc x \cdot \tan x} \\ &= \frac{2\tan x}{\csc x \cdot \tan x} \\ &= \frac{2}{\csc x} \\ &= \frac{2}{\frac{1}{\sin x}} \\ &= 2\sin x\end{aligned}$$

5-1 Trigonometric Identities

27. $\dfrac{\csc x\cos x+\cot x}{\sec x\cot x}$

SOLUTION:

$$\begin{aligned}\frac{\csc x\cos x+\cot x}{\sec x\cot x}&=\frac{\frac{1}{\sin x}\cos x+\frac{\cos x}{\sin x}}{\frac{1}{\cos x}\cdot\frac{\cos x}{\sin x}}\\&=\frac{\frac{\cos x}{\sin x}+\frac{\cos x}{\sin x}}{\frac{1}{\sin x}}\\&=\frac{\frac{2\cos x}{\sin x}}{\frac{1}{\sin x}}\\&=\frac{2\cos x}{\sin x}\cdot\sin x\\&=2\cos x\end{aligned}$$

29. $\dfrac{\sec^2 x}{\cot^2 x+1}$

SOLUTION:

$$\begin{aligned}\frac{\sec^2 x}{\cot^2 x+1}&=\frac{\sec^2 x}{\csc^2 x}\\&=\frac{\frac{1}{\cos^2 x}}{\frac{1}{\sin^2 x}}\\&=\frac{1}{\cos^2 x}\cdot\sin^2 x\\&=\frac{\sin^2 x}{\cos^2 x}\\&=\tan^2 x\end{aligned}$$

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31. $\cot x - \cos^3 x \csc x$

SOLUTION:

$$\begin{aligned}\cot x - \cos^3 x \csc x &= \frac{\cos x}{\sin x} - \cos^3 x \csc x \\ &= \frac{\cos x}{\sin x} - \cos^3 x \cdot \frac{1}{\sin x} \\ &= \frac{\cos x}{\sin x} - \frac{\cos^3 x}{\sin x} \\ &= \frac{\cos x - \cos^3 x}{\sin x} \\ &= \frac{(1-\cos^2 x)\cos x}{\sin x} \\ &= \frac{(\sin^2 x)\cos x}{\sin x} \\ &= \sin x \cos x\end{aligned}$$

Simplify each expression.

33. $\dfrac{1-\cos x}{\tan x} + \dfrac{\sin x}{1+\cos x}$

SOLUTION:

$$\begin{aligned}&\frac{1-\cos x}{\tan x} + \frac{\sin x}{1+\cos x} \\ &= \frac{(1-\cos x)(1+\cos x)}{\tan x(1+\cos x)} + \frac{\tan x \sin x}{\tan x(1+\cos x)} \\ &= \frac{1-\cos^2 x}{\tan x(1+\cos x)} + \frac{\tan x \sin x}{\tan x(1+\cos x)} \\ &= \frac{\sin^2 x + \tan x \sin x}{\tan x(1+\cos x)} \\ &= \frac{\sin x(\sin x + \tan x)}{\tan x(1+\cos x)} \\ &= \frac{\sin x(\sin x + \tan x)}{\tan x + \tan x \cos x} \\ &= \frac{\sin x(\sin x + \tan x)}{\tan x + \frac{\sin x}{\cos x} \cdot \cos x} \\ &= \frac{\sin x(\sin x + \tan x)}{\tan x + \sin x} \\ &= \frac{\sin x(\sin x + \tan x)}{\sin x + \tan x} \\ &= \sin x\end{aligned}$$

5-1 Trigonometric Identities

35. $\dfrac{\cos x \cot x}{\sec x + \tan x} + \dfrac{\sin x}{\sec x - \tan x}$

SOLUTION:

$$\frac{\cos x \cot x}{\sec x + \tan x} + \frac{\sin x}{\sec x - \tan x}$$

$$= \frac{\cos x \cot x(\sec x - \tan x)}{(\sec x + \tan x)(\sec x - \tan x)} + \frac{\sin x(\sec x + \tan x)}{(\sec x + \tan x)(\sec x - \tan x)}$$

$$= \frac{\cos x \cot x(\sec x - \tan x) + \sin x(\sec x + \tan x)}{(\sec x + \tan x)(\sec x - \tan x)}$$

$$= \frac{\cos x \cot x \sec x - \cos x \cot x \tan x + \sin x \sec x + \sin x \tan x}{\sec^2 x - \tan^2 x}$$

$$= \frac{\cos x \cot x \frac{1}{\cos x} - \cos x \frac{1}{\tan x} \tan x + \sin x \frac{1}{\cos x} + \sin x \tan x}{\sec^2 x - \tan^2 x}$$

$$= \frac{\cot x - \cos x + \tan x + \sin x \tan x}{1}$$

$$= \cot x - \cos x + \tan x + \sin x \tan x$$

Rewrite as an expression that does not involve a fraction.

39. $\dfrac{\csc x}{1 - \sin x}$

SOLUTION:

$$\frac{\csc x}{1 - \sin x} = \frac{\csc x}{1 - \sin x} \cdot \frac{1 + \sin x}{1 + \sin x}$$

$$= \frac{\csc x(1 + \sin x)}{(1 - \sin x)(1 + \sin x)}$$

$$= \frac{\csc x + \csc x \sin x}{1 - \sin^2 x}$$

$$= \frac{\csc x + \frac{1}{\sin x} \sin x}{1 - \sin^2 x}$$

$$= \frac{\csc x + 1}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x} \cdot \frac{\csc x + 1}{1}$$

$$= \sec^2 x \cdot \frac{\csc x + 1}{1}$$

$$= \sec^2 x(\csc x + 1)$$

5-1 Trigonometric Identities

41. $\dfrac{\cot x}{1+\sin x}$

SOLUTION:

$$\begin{aligned}\frac{\cot x}{1+\sin x} &= \frac{\cot x}{1+\sin x}\cdot\frac{1-\sin x}{1-\sin x}\\ &= \frac{\cot x(1-\sin x)}{(1+\sin x)(1-\sin x)}\\ &= \frac{\cot x(1-\sin x)}{1-\sin^2 x}\\ &= \frac{\cot x(1-\sin x)}{\cos^2 x}\\ &= \frac{\cot x}{\cos^2 x}(1-\sin x)\\ &= \frac{\cos x}{\sin x}\cdot\frac{1}{\cos^2 x}(1-\sin x)\\ &= \frac{1}{\sin x\cos x}(1-\sin x)\\ &= \frac{1}{\sin x\cos x}-\frac{\sin x}{\sin x\cos x}\\ &= \frac{1}{\sin x}\cdot\frac{1}{\cos x}-\frac{1}{\cos x}\\ &= \csc x\sec x-\sec x\\ &= \sec x(\csc x-1)\end{aligned}$$

43. $\dfrac{2\sin x}{\cot x+\csc x}$

SOLUTION:

$$\begin{aligned}\frac{2\sin x}{\cot x+\csc x} &= \frac{2\sin x}{\cot x+\csc x}\cdot\frac{\cot x-\csc x}{\cot x-\csc x}\\ &= \frac{2\sin x(\cot x-\csc x)}{(\cot x+\csc x)(\cot x-\csc x)}\\ &= \frac{2\sin x\cot x-2\sin x\csc x}{\cot^2 x-\csc^2 x}\\ &= \frac{2\sin x\cot x-2\sin x\csc x}{-1}\\ &= -2\sin x\cot x+2\sin x\csc x\\ &= -2\sin x\frac{\cos x}{\sin x}+2\sin x\frac{1}{\sin x}\\ &= -2\cos x+2\\ &= 2-2\cos x\end{aligned}$$

45. $\dfrac{\cot^2 x \cos x}{\csc x - 1}$

SOLUTION:

$$\frac{\cot^2 x \cos x}{\csc x - 1} = \frac{\cot^2 x \cos x}{\csc x - 1} \cdot \frac{\csc x + 1}{\csc x + 1}$$
$$= \frac{\cot^2 x \cos x(\csc x + 1)}{(\csc x - 1)(\csc x + 1)}$$
$$= \frac{\cot^2 x \cos x(\csc x + 1)}{\csc^2 x - 1}$$
$$= \frac{\cot^2 x \cos x(\csc x + 1)}{\cot^2 x}$$
$$= \cos x(\csc x + 1)$$

47. $\dfrac{\sin x \tan x}{\cos x + 1}$

SOLUTION:

$$\frac{\sin x \tan x}{\cos x + 1} = \frac{\sin x \tan x}{\cos x + 1} \cdot \frac{\cos x - 1}{\cos x - 1}$$
$$= \frac{\sin x \tan x(\cos x - 1)}{(\cos x + 1)(\cos x - 1)}$$
$$= \frac{\sin x \tan x(\cos x - 1)}{\cos^2 x - 1}$$
$$= \frac{\sin x \tan x(\cos x - 1)}{-(1 - \cos^2 x)}$$
$$= \frac{\sin x \tan x(\cos x - 1)}{-(\sin^2 x)}$$
$$= \frac{\tan x(\cos x - 1)}{-\sin x}$$
$$= -\frac{\sin x}{\cos x} \cdot \frac{\cos x - 1}{\sin x}$$
$$= -\frac{\cos x - 1}{\cos x}$$
$$= \frac{1 - \cos x}{\cos x}$$
$$= \frac{1}{\cos x} - \frac{\cos x}{\cos x}$$
$$= \sec x - 1$$

5-1 Trigonometric Identities

Write each expression in terms of a single trigonometric function.

51. $\tan x - \csc x \sec x$

SOLUTION:

$$\begin{aligned}\tan x - \csc x \sec x &= \frac{\sin x}{\cos x} - \frac{1}{\sin x} \cdot \frac{1}{\cos x} \\ &= \frac{\sin x}{\cos x} - \frac{1}{\sin x \cos x} \\ &= \frac{\sin^2 x}{\sin x \cos x} - \frac{1}{\sin x \cos x} \\ &= \frac{\sin^2 x - 1}{\sin x \cos x} \\ &= \frac{-\cos^2 x}{\sin x \cos x} \\ &= -\frac{\cos x}{\sin x} \\ &= -\cot x\end{aligned}$$

53. $\csc x \tan^2 x - \sec^2 x \csc x$

SOLUTION:

$$\begin{aligned}&\csc x \tan^2 x - \sec^2 x \csc x \\ &= \frac{1}{\sin x} \cdot \frac{\sin^2 x}{\cos^2 x} - \frac{1}{\cos^2 x} \cdot \frac{1}{\sin x} \\ &= \frac{\sin^2 x}{\sin x \cos^2 x} - \frac{1}{\cos^2 x \sin x} \\ &= \frac{\sin^2 x - 1}{\sin x \cos^2 x} \\ &= \frac{-\cos^2 x}{\sin x \cos^2 x} \\ &= \frac{-1}{\sin x} \\ &= -\csc x\end{aligned}$$