4-2 Degrees and Radians

Write each decimal degree measure in DMS form and each DMS measure in decimal degree form to the nearest thousandth.

7. 45° 21′ 25″

SOLUTION:

Each minute is $\frac{1}{60}$ of a degree and each second is $\frac{1}{60}$ of a minute, so each second is $\frac{1}{3600}$ of a degree.

$$\begin{aligned} 45^\circ\ 21'\ 25' &= 45^\circ + 21'\left(\frac{1^\circ}{60'}\right) + 25''\left(\frac{1^\circ}{3600''}\right) \\ &\approx 45^\circ + 0.35^\circ + 0.007 \\ &\approx 45.357^\circ \end{aligned}$$

Therefore, 45° 21′ 25″ can be written as about 45.357°.

Write each degree measure in radians as a multiple of π and each radian measure in degrees.

11. 225°

SOLUTION:

To convert a degree measure to radians, multiply by $\frac{\pi \text{ radians}}{180^\circ}$.

$$\begin{aligned} 225^\circ &= 225^\circ\left(\frac{\pi \text{ radians}}{180^\circ}\right) \\ &= \frac{5\pi}{4} \text{ radians} \\ &= \frac{5\pi}{4} \end{aligned}$$

13. –45°

SOLUTION:

To convert a degree measure to radians, multiply by $\frac{\pi \text{ radians}}{180^\circ}$.

$$\begin{aligned} -45^\circ &= -45^\circ\left(\frac{\pi \text{ radians}}{180^\circ}\right) \\ &= -\frac{\pi}{4} \text{ radians} \\ &= -\frac{\pi}{4} \end{aligned}$$

15. $\frac{5\pi}{2}$

SOLUTION:

To convert a radian measure to degrees, multiply by $\frac{180°}{\pi \text{ radians}}$.

$$\begin{aligned}\frac{5\pi}{2} &= \frac{5\pi}{2} \text{ radians} \\ &= \frac{5\pi}{2} \text{ radians}\left(\frac{180°}{\pi \text{ radians}}\right) \\ &= \frac{900°}{2} \\ &= 450°\end{aligned}$$

17. $-\frac{7\pi}{6}$

SOLUTION:

To convert a radian measure to degrees, multiply by $\frac{180°}{\pi \text{ radians}}$.

$$\begin{aligned}-\frac{7\pi}{6} &= -\frac{7\pi}{6} \text{ radians} \\ &= -\frac{7\pi}{6} \text{ radians}\left(\frac{180°}{\pi \text{ radians}}\right) \\ &= -\frac{1260}{6} \quad -210° \\ &= -210°\end{aligned}$$

4-2 Degrees and Radians

Identify all angles that are coterminal with the given angle. Then find and draw one positive and one negative angle coterminal with the given angle.

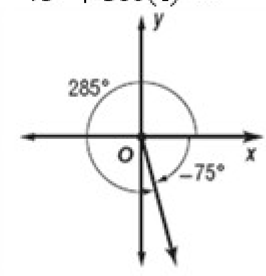
19. –75°

SOLUTION:

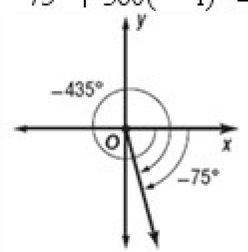
All angles measuring $-75° + 360n°$ are coterminal with a $-75°$ angle.

Sample answer: Let n = 1 and −1.

$$-75° + 360(1)° = -75° + 360° = 285°$$



$$-75° + 360(-1)° = -75° - 360° = -435°$$



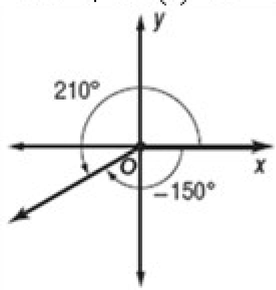
21. –150°

SOLUTION:

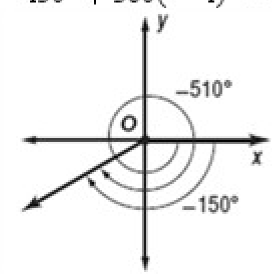
All angles measuring $-150° + 360n°$ are coterminal with a $-150°$ angle.

Sample answer: Let *n* = 1 and −1.

$-150° + 360(1)° = -150° + 360° = 210°$



$-150° + 360(-1)° = -150° - 360° = -510°$



Find the length of the intercepted arc with the given central angle measure in a circle with the given radius. Round to the nearest tenth.

27. $\frac{\pi}{6}$, *r* = 2.5 m

SOLUTION:

$$\begin{aligned} s &= r\theta \\ &= 2.5\left(\frac{\pi}{6}\right) \\ &= \frac{5\pi}{12} \\ &\approx 1.3 \end{aligned}$$

29. $\frac{5\pi}{12}$, *r* = 4 yd

SOLUTION:

$$\begin{aligned} s &= r\theta \\ &= 4\left(\frac{5\pi}{12}\right) \\ &= \frac{20\pi}{12} \\ &\approx 5.2 \end{aligned}$$

4-2 Degrees and Radians

31. 45°, r = 5 mi

SOLUTION:

Method 1

Convert 45° to radian measure, and then use $s = r\theta$ to find the arc length.

$$45° = 45°\left(\frac{\pi \text{ radians}}{180°}\right)$$
$$= \frac{\pi}{4}$$

Substitute r = 5 and $\theta = \frac{\pi}{4}$.

$$s = r\theta$$
$$= 5\left(\frac{\pi}{4}\right)$$
$$= \frac{5\pi}{4}$$
$$\approx 3.9$$

Method 2

Use $s = \frac{\pi r\theta}{180°}$ to find the arc length.

$$s = \frac{\pi r\theta}{180°}$$
$$= \frac{\pi(5)(45°)}{180°}$$
$$= \frac{5\pi}{4}$$
$$\approx 3.9$$

Find the rotation in revolutions per minute given the angular speed and the radius given the linear speed and the rate of rotation.

35. $\omega = 135\pi$ rad/h

SOLUTION:

The angular speed is 135π radians per hour.

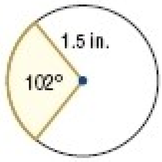
$$\omega = \frac{\theta}{t}$$
$$\frac{135\pi \text{ rad}}{1\text{ h}} = \frac{\theta}{1\text{ min}}$$
$$\frac{1\text{ h}}{60\text{ min}} \cdot \frac{135\pi \text{ rad}}{1\text{ h}} = \frac{\theta}{1\text{ min}}$$
$$\frac{\frac{135\pi}{60}\text{ rad}}{1\text{ min}} = \frac{\theta}{1\text{ min}}$$
$$2.25\pi \text{ rad} = \theta$$

Each revolution measures 2π radians.

$2.25\pi \div 2\pi = 1.125$

The angle of rotation is 1.125 revolutions per minute.

Find the area of each sector.

43. 

SOLUTION:

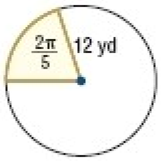
The measure of the sector's central angle θ is 102° and the radius is 1.5 inches. Convert the central angle measure to radians.

$$102° = 102°\left(\frac{\pi \text{ radians}}{180°}\right) = \frac{17\pi}{30}$$

Use the central angle and the radius to find the area of the sector.

$$A = \frac{1}{2}r^2\theta = \frac{1}{2}(1.5)^2\left(\frac{17\pi}{30}\right) \approx 2.0$$

Therefore, the area of the sector is about 2.0 square inches.

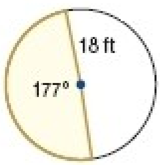
45. 

SOLUTION:

The measure of the sector's central angle θ is $\frac{2\pi}{5}$ and the radius is 12 yards.

$$\begin{aligned} A &= \frac{1}{2}r^2\theta \\ &= \frac{1}{2}(12)^2\left(\frac{2\pi}{5}\right) \\ &\approx 90.5 \end{aligned}$$

Therefore, the area of the sector is about 90.5 square yards.

47. 

SOLUTION:

The measure of the sector's central angle θ is 177° and the radius is 18 feet. Convert the central angle measure to radians.

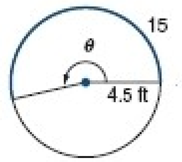
$$\begin{aligned} 177° &= 177°\left(\frac{\pi \text{ radians}}{180°}\right) \\ &= \frac{59\pi}{60} \end{aligned}$$

Use the central angle and the radius to find the area of the sector.

$$\begin{aligned} A &= \frac{1}{2}r^2\theta \\ &= \frac{1}{2}(18)^2\left(\frac{59\pi}{60}\right) \\ &\approx 500.5 \end{aligned}$$

Therefore, the area of the sector is about 500.5 square feet.

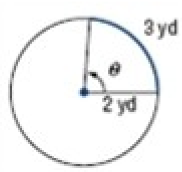
Find the measure of angle θ in radians and degrees.

59. 

SOLUTION:

$$s = r\theta$$
$$15 = 4.5\theta$$
$$3.333 \approx \theta$$

So, $\theta \approx 3.3$ radians, or $3.333\left(\frac{180°}{\pi \text{ radians}}\right) \approx 191°$.

61. 

SOLUTION:

$$s = r\theta$$
$$3 = 2\theta$$
$$1.5 = \theta$$

So, $\theta = 1.5$ radians, or $1.5\left(\frac{180°}{\pi \text{ radians}}\right) \approx 85.9°$.