4-3 Trigonometric Functions on the Unit Circle

The given point lies on the terminal side of an angle θ in standard position. Find the values of the six trigonometric functions of θ.

1. (3, 4)

SOLUTION:

Use the values of x and y to find r.

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ &= \sqrt{3^2 + 4^2} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

Use $x = 3$, $y = 4$, and $r = 5$ to write the six trigonometric ratios.

$$\sin\theta = \frac{y}{r} = \frac{4}{5} \qquad \cos\theta = \frac{x}{r} = \frac{3}{5} \qquad \tan\theta = \frac{y}{x} = \frac{4}{3}$$

$$\csc\theta = \frac{r}{y} = \frac{5}{4} \qquad \sec\theta = \frac{r}{x} = \frac{5}{3} \qquad \cot\theta = \frac{x}{y} = \frac{3}{4}$$

3. (−4, −3)

SOLUTION:

Use the values of x and y to find r.

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ &= \sqrt{(-4)^2 + (-3)^2} \\ &= \sqrt{25} \text{ or } 5 \end{aligned}$$

Use $x = -4$, $y = -3$, and $r = 5$ to write the six trigonometric ratios.

$$\sin\theta = \frac{y}{r} = -\frac{3}{5} \qquad \cos\theta = \frac{x}{r} = -\frac{4}{5} \qquad \tan\theta = \frac{y}{x} = \frac{3}{4}$$

$$\csc\theta = \frac{r}{y} = -\frac{5}{3} \qquad \sec\theta = \frac{r}{x} = -\frac{5}{4} \qquad \cot\theta = \frac{x}{y} = \frac{4}{3}$$

5. (1, −8)

SOLUTION:

Use the values of x and y to find r.

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ &= \sqrt{1^2 + (-8)^2} \\ &= \sqrt{65} \end{aligned}$$

Use $x = 1, y = -8$, and $r = \sqrt{65}$ to write the six trigonometric ratios.

$$\sin\theta = \frac{y}{r} = -\frac{8}{\sqrt{65}} = -\frac{8\sqrt{65}}{65}$$

$$\csc\theta = \frac{r}{y} = -\frac{\sqrt{65}}{8}$$

$$\tan\theta = \frac{y}{x} = -8$$

$$\cos\theta = \frac{x}{r} = -\frac{1}{\sqrt{65}} = -\frac{\sqrt{65}}{65}$$

$$\sec\theta = \frac{r}{x} = \frac{\sqrt{65}}{1} = \sqrt{65}$$

$$\cot\theta = \frac{x}{y} = -\frac{1}{8}$$

7. (−8, 15)

SOLUTION:

Use the values of x and y to find r.

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ &= \sqrt{(-8)^2 + 15^2} \\ &= \sqrt{64 + 225} = \sqrt{289} \text{ or } 17 \end{aligned}$$

Use $x = -8, y = 15$, and $r = 17$ to write the six trigonometric ratios.

$$\sin\theta = \frac{y}{r} = \frac{15}{17} \qquad \cos\theta = \frac{x}{r} = -\frac{8}{17} \qquad \tan\theta = \frac{y}{x} = -\frac{15}{8}$$

$$\csc\theta = \frac{r}{y} = \frac{17}{15} \qquad \sec\theta = \frac{r}{x} = -\frac{17}{8} \qquad \cot\theta = \frac{x}{y} = -\frac{8}{15}$$

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Find the exact value of each trigonometric function, if defined. If not defined, write *undefined*.

9. $\sin \frac{\pi}{2}$

SOLUTION:

The terminal side of $\frac{\pi}{2}$ in standard position lies on the positive y-axis. Choose a point $P(0, 1)$ on the terminal side of the angle because $r = 1$.

$$\begin{aligned}\sin \frac{\pi}{2} &= \frac{y}{r} \\ &= \frac{1}{1} \text{ or } 1\end{aligned}$$

11. cot (–180°)

SOLUTION:

The terminal side of $-180°$ in standard position lies on the negative x-axis. Choose a point $P(-1, 0)$ on the terminal side of the angle because $r = 1$.

$$\begin{aligned}\cot(-180°) &= \frac{x}{y} \\ &= \frac{-1}{0} \text{ or undefined}\end{aligned}$$

13. cos (–270°)

SOLUTION:

The terminal side of $-270°$ in standard position lies on the positive y-axis. Choose a point $P(0, 1)$ on the terminal side of the angle because $r = 1$.

$$\begin{aligned}\cos(-270°) &= \frac{x}{r} \\ &= \frac{0}{1} \text{ or } 0\end{aligned}$$

15. tan π

SOLUTION:

The terminal side of π in standard position lies on the negative x-axis. Choose a point $P(-1, 0)$ on the terminal side of the angle because $r = 1$.

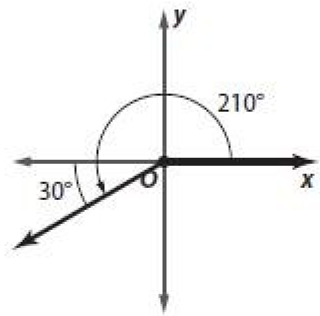
$$\begin{aligned}\tan \pi &= \frac{y}{x} \\ &= \frac{0}{-1} \text{ or } 0\end{aligned}$$

Sketch each angle. Then find its reference angle.

17. 210°

SOLUTION:

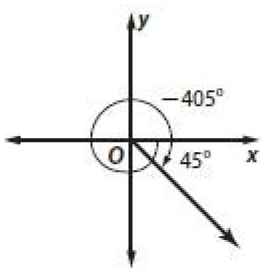
The terminal side of 210º lies in Quadrant III. Therefore, its reference angle is $\theta' = 210º – 180º$ or 30º.



21. –405°

SOLUTION:

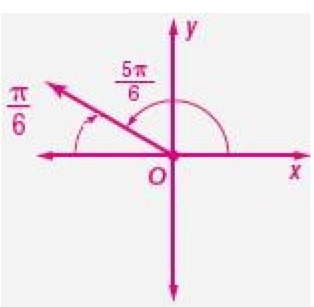
A coterminal angle is –405° + 360(2)° or 315°. The terminal side of 315° lies in Quadrant IV, so its reference angle is 360º – 315º or 45º.



23. $\frac{5\pi}{6}$

SOLUTION:

The terminal side of $\frac{5\pi}{6}$ lies in Quadrant II. Therefore, its reference angle is $\theta' = \pi - \frac{5\pi}{6}$ or $\frac{\pi}{6}$.



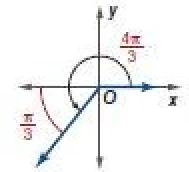
4-3 Trigonometric Functions on the Unit Circle

Find the exact value of each expression.

25. $\cos \frac{4\pi}{3}$

SOLUTION:

Because the terminal side of θ lies in Quadrant III, the reference angle θ' is $\frac{4\pi}{3} - \pi$ or $\frac{\pi}{3}$.



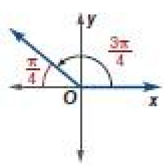
In Quadrant III, $\cos \theta$ is negative and $\cos\frac{\pi}{3} = \frac{1}{2}$

$$\cos\frac{4\pi}{3} = -\cos\frac{\pi}{3}$$
$$= -\frac{1}{2}$$

27. $\sin \frac{3\pi}{4}$

SOLUTION:

Because the terminal side of θ lies in Quadrant II, the reference angle θ' is $\pi - \frac{3\pi}{4}$ or $\frac{\pi}{4}$.



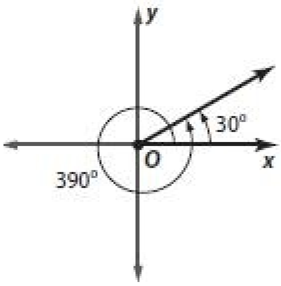
In Quadrant II, $\sin \theta$ is positive and $\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$.

$$\sin\frac{3\pi}{4} = \sin\frac{\pi}{4}$$
$$= \frac{\sqrt{2}}{2}$$

29. csc 390°

SOLUTION:

A coterminal angle is 390° + 360° or 30°, which lies in Quadrant I. So, the reference angle θ ' is 360° − 30° or 30°. Because sine and cosecant are reciprocal functions and sin θ is positive in Quadrant I, it follows that csc θ is also positive in Quadrant I.

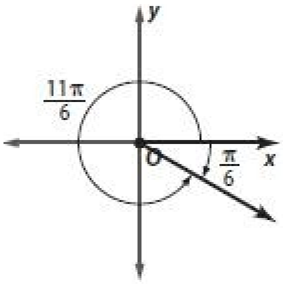


$$\begin{aligned}\csc 390° &= \csc 30° \\ &= \frac{1}{\sin 30°} \\ &= \frac{1}{\frac{1}{2}} \text{ or } 2\end{aligned}$$

31. $\tan \frac{11\pi}{6}$

SOLUTION:

Because the terminal side of θ lies in Quadrant IV, the reference angle θ' is $2\pi - \frac{11\pi}{6}$ or $\frac{\pi}{6}$. In Quadrant IV, $\tan \theta$ is negative.



$$\begin{aligned}\tan\frac{11\pi}{6} &= -\tan\frac{\pi}{6}\\ &= \frac{-\sin\frac{\pi}{6}}{\cos\frac{\pi}{6}}\\ &= \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}}\\ &= \frac{-1}{\sqrt{3}}\\ &= -\frac{\sqrt{3}}{3}\end{aligned}$$