4-1 Right Triangle Trigonometry

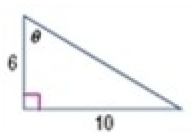
Find the measure of angle θ. Round to the nearest degree, if necessary.

31. 

SOLUTION:

Because the lengths of the sides opposite θ and the hypotenuse are given, the sine function can be used to find θ.

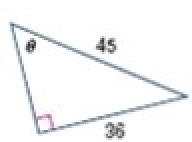
$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$
$$\sin\theta = \frac{7}{29}$$
$$\theta = \sin^{-1}\frac{7}{29}$$
$$\theta \approx 14$$

35. 

SOLUTION:

Because the lengths of the sides opposite and adjacent to θ are given, the tangent function can be used to find θ.

$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$
$$\tan\theta = \frac{10}{6}$$
$$\theta = \tan^{-1}\frac{10}{6} \text{ or about } 59°$$

37. 

SOLUTION:

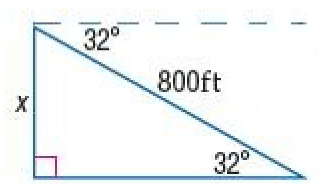
Because the length of the hypotenuse and side opposite θ are given, the sine function can be used to find θ.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$
$$\sin\theta = \frac{36}{45}$$
$$\theta = \sin^{-1}\frac{36}{45} \text{ or about } 53°$$

39. **PARASAILING** Kayla decided to try parasailing. She was strapped into a parachute towed by a boat. An 800-foot line connected her parachute to the boat, which was at a 32º angle of depression below her. How high above the water was Kayla?

SOLUTION:

The angle of elevation from the boat to the parachute is equivalent to the angle of depression from the parachute to the boat because the two angles are alternate interior angles, as shown below.



Because an acute angle and the hypotenuse are given, the sine function can be used to find *x*.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$
$$\sin 32° = \frac{x}{800}$$
$$800 \sin 32° = x$$
$$423.9 \approx x$$

Therefore, Kayla was about 424 feet above the water.

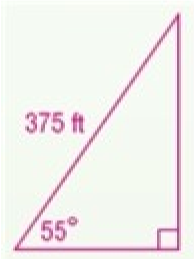
41. **ROLLER COASTER** On a roller coaster, 375 feet of track ascend at a 55º angle of elevation to the top before the first and highest drop.

a. Draw a diagram to represent the situation.

b. Determine the height of the roller coaster.

SOLUTION:

a. Draw a diagram of a right triangle. Since the track is ascending at a 55º, the track is represented by the hypotenuse. Label the hypotenuse 375. Then label the acute angle 55º .

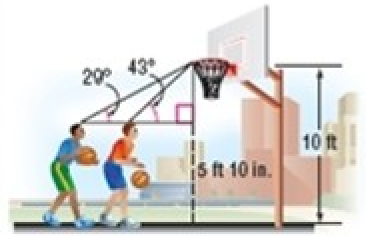


b. Because an acute angle and the length of the hypotenuse are given, you can use the sine function to find the length of the opposite side.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$
$$\sin 55° = \frac{x}{375}$$
$$375 \sin 55° = x$$
$$307.2 \approx x$$

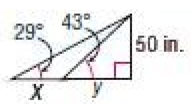
Therefore, the height of the roller coaster is about 307 feet.

43. **BASKETBALL** Both Derek and Sam are 5 feet 10 inches tall. Derek looks at a 10-foot basketball goal with an angle of elevation of 29°, and Sam looks at the goal with an angle of elevation of 43°. If Sam is directly in front of Derek, how far apart are the boys standing?



SOLUTION:

Draw a diagram to model the situation. The vertical distance from the boys' heads to the rim is 10(12) – [5(12) + 10] = 50 inches. Label the horizontal distance between Sam and Derek as x and the horizontal distance between Sam and the goal as y.



From the smaller right triangle, you can use the tangent function to find y.

$$\begin{aligned} \tan\theta &= \frac{\text{opp}}{\text{adj}} \\ \tan 43° &= \frac{50}{y} \\ y\tan 43° &= 50 \\ y &= \frac{50}{\tan 43°} \end{aligned}$$

From the larger right triangle, you can use the tangent function to find x.

$$\begin{aligned} \tan\theta &= \frac{\text{opp}}{\text{adj}} \\ \tan 29° &= \frac{50}{x+y} \\ (x+y)\tan 29° &= 50 \\ (x+y) &= \frac{50}{\tan 29°} \\ x + \frac{50}{\tan 43°} &= \frac{50}{\tan 29°} \\ x &= \frac{50}{\tan 29°} - \frac{50}{\tan 43°} \\ x &\approx 36.6 \end{aligned}$$

Therefore, Derek and Sam are standing about 36.6 inches ≈ 3.1 feet apart.

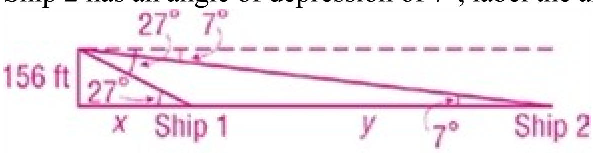
45. **LIGHTHOUSE** Two ships are spotted from the top of a 156-foot lighthouse. The first ship is at a 27º angle of depression, and the second ship is directly behind the first at a 7º angle of depression.

a. Draw a diagram to represent the situation.

b. Determine the distance between the two ships.

SOLUTION:

a. Draw two right triangle with common height and base on the same line. The light house is 156 ft, label the height 156. Since Ship 1 has an angle of depression of 27°, label the angle opposite 156 ft in the smaller triangle 27°. Since Ship 2 has an angle of depression of 7°, label the angle opposite 156 ft in the larger triangle 7°..



b. From the smaller right triangle, you can use the tangent function to find x.

$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$
$$\tan 27° = \frac{156}{x}$$
$$x\tan 27° = 156$$
$$x = \frac{156}{\tan 27°}$$

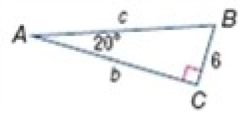
From the larger right triangle, you can use the tangent function to find y.

$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$
$$\tan 7° = \frac{156}{x+y}$$
$$(x+y)\tan 7° = 156$$
$$x+y = \frac{156}{\tan 7°}$$
$$\frac{156}{\tan 27°} + y = \frac{156}{\tan 7°}$$
$$y = \frac{156}{\tan 7°} - \frac{156}{\tan 27°}$$
$$y \approx 964.4$$

Therefore, the distance between the two ships is about 964 feet.

4-1 Right Triangle Trigonometry

Solve each triangle. Round side lengths to the nearest tenth and angle measures to the nearest degree.

47. 

SOLUTION:

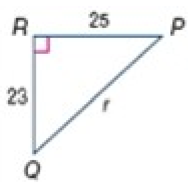
Use trigonometric functions to find *b* and *c*.

$$\tan 20° = \frac{6}{b} \qquad \sin 20° = \frac{6}{c}$$
$$b \tan 20° = 6 \qquad c \sin 20° = 6$$
$$b = \frac{6}{\tan 20°} \qquad c = \frac{6}{\sin 20°}$$
$$b \approx 16.5 \qquad c \approx 17.5$$

Because the measures of two angles are given, *B* can be found by subtracting *A* from $90°$.

$$20° + B = 90°$$
$$B = 70°$$

Therefore, $B = 70°$, $b \approx 16.5$, $c \approx 17.5$.

49. 

SOLUTION:

Use the Pythagorean Theorem to find *r*.

$$r = \sqrt{23^2 + 25^2}$$
$$\approx 34.0$$

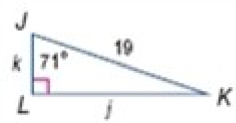
Use the tangent function to find *P*.

$$\tan P = \frac{23}{25}$$
$$P = \tan^{-1} \frac{23}{25}$$
$$P \approx 42.61°$$

Because the measures of two angles are now known, you can find *Q* by subtracting *P* from $90°$.

$$42.61° + Q \approx 90°$$
$$Q \approx 47.39°$$

Therefore, $P \approx 43°$, $Q \approx 47°$, and $r \approx 34.0$.

51. 

SOLUTION:

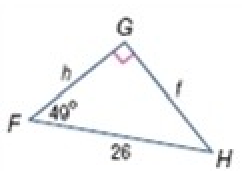
Use trigonometric functions to find j and k.

$$\sin 71^\circ = \frac{j}{19} \qquad \cos 71^\circ = \frac{k}{19}$$
$$19 \sin 71^\circ = j \qquad 19 \cos 71^\circ = k$$
$$18.0 \approx j \qquad 6.2 \approx k$$

Because the measures of two angles are given, K can be found by subtracting J from 90°.

$$71^\circ + K \approx 90^\circ$$
$$K \approx 19^\circ$$

Therefore, $K \approx 19^\circ, j \approx 18.0, k \approx 6.2$.

53. 

SOLUTION:

Use trigonometric functions to find f and h.

$$\sin 49^\circ = \frac{f}{26} \qquad \cos 49^\circ = \frac{h}{26}$$
$$26 \sin 49^\circ = f \qquad 26 \cos 49^\circ = h$$
$$19.6 \approx f \qquad 17.1 \approx h$$

Because the measures of two angles are given, H can be found by subtracting F from 90°.

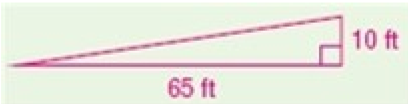
$$49^\circ + H = 90^\circ$$
$$H = 41^\circ$$

Therefore, $H = 41^\circ, f \approx 19.6, h \approx 17.1$.

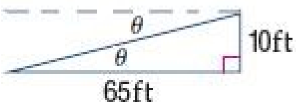
55. **BASEBALL** Michael's seat at a game is 65 feet behind home plate. His line of vision is 10 feet above the field.
a. Draw a diagram to represent the situation.
b. What is the angle of depression to home plate?

SOLUTION:

a. Draw a right triangle. Michael's seat at a game is 65 feet behind home plate, which represents the horizontal distance. Label the bottom of the triangle 65 ft. Michael's line of vision is the vertical distance. Label the vertical side 10 feet.



b. Michael's angle of depression to home plate is equivalent to the angle of elevation from home plate to where he is sitting.



Use the tangent function to find *θ*.

$$\tan\theta = \frac{10}{65}$$
$$\theta = \tan^{-1}\frac{10}{65}$$
$$\theta \approx 8.7°$$

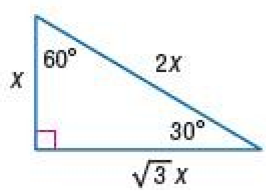
Therefore, the angle of depression to home plate is about $9°$.

Find the exact value of each expression without using a calculator.

57. sin 60°

SOLUTION:

Draw a diagram of a 30º-60º-90º triangle.



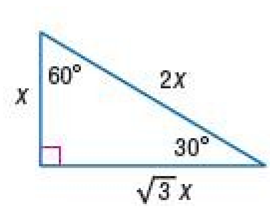
The length of the side opposite the 60° angle is $\sqrt{3}\,x$ and the length of the hypotenuse is 2*x*.

$$\sin 60° = \frac{\text{opp}}{\text{hyp}} = \frac{\sqrt{3}x}{2x} = \frac{\sqrt{3}}{2}$$

59. sec 30°

SOLUTION:

Draw a diagram of a 30º-60º-90º triangle.



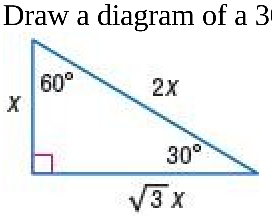
The length of the hypotenuse is 2*x* and the length of the side adjacent to the 30º angle is $\sqrt{3}\,x$.

$$\sec 30° = \frac{\text{hyp}}{\text{adj}} = \frac{2x}{\sqrt{3}x} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

61. tan 60°

SOLUTION:

Draw a diagram of a 30º-60º-90º triangle.



The length of the side opposite the 60º is $\sqrt{3}\,x$ and the length of the adjacent side is *x*.

$$\tan 60° = \frac{\text{opp}}{\text{adj}} = \frac{\sqrt{3}x}{x} = \sqrt{3}$$

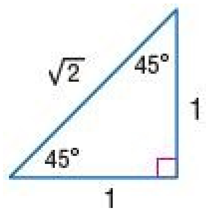
4-1 Right Triangle Trigonometry

Without using a calculator, find the measure of the acute angle θ in a right triangle that satisfies each equation.

63. $\tan\theta = 1$

SOLUTION:

Because $\tan\theta = 1$ and $\tan\theta = \frac{\text{opp}}{\text{adj}}$, it follows that $\frac{\text{opp}}{\text{adj}} = 1$. In the $45°\text{-}45°\text{-}90°$ triangle below, the side length opposite an acute angle is 1 and the adjacent side length is also 1. So, $\frac{\text{opp}}{\text{adj}} = 1$.

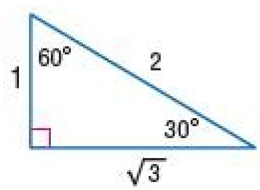


Therefore, $\theta = 45°$.

65. $\cot\theta = \frac{\sqrt{3}}{3}$

SOLUTION:

Because $\cot\theta = \frac{\sqrt{3}}{3}$ and $\cot\theta = \frac{\text{adj}}{\text{opp}}$, it follows that $\frac{\text{adj}}{\text{opp}} = \frac{\sqrt{3}}{3}$. In the $30°\text{-}60°\text{-}90°$ triangle below, the side length that is adjacent to the 60º angle is 1 and the length of the opposite side is $\sqrt{3}$. So, $\frac{\text{adj}}{\text{opp}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$.



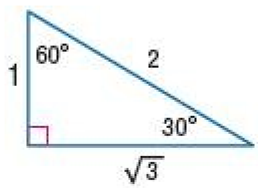
Therefore, $\theta = 60°$.

4-1 Right Triangle Trigonometry

67. csc $\theta = 2$

SOLUTION:

Because csc $\theta = 2$ and csc $\theta = \frac{\text{hyp}}{\text{opp}}$, it follows that $\frac{\text{hyp}}{\text{opp}} = 2 = \frac{2}{1}$. In the $30°\text{-}60°\text{-}90°$ triangle below, the hypotenuse is 2 and the side length that is opposite the 30º angle is 1. So, $\frac{\text{hyp}}{\text{opp}} = \frac{2}{1} = 2$.



Therefore, $\theta = 30°$.