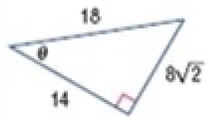
4-1 Right Triangle Trigonometry

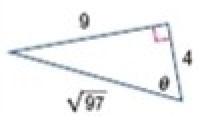
Find the exact values of the six trigonometric functions of *θ*.

1. 

SOLUTION:

The length of the side opposite θ is $8\sqrt{2}$, the length of the side adjacent to θ is 14, and the length of the hypotenuse is 18.

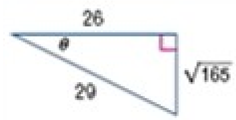
$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{8\sqrt{2}}{18} \text{ or } \frac{4\sqrt{2}}{9}$$
$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{14}{18} \text{ or } \frac{7}{9}$$
$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{8\sqrt{2}}{14} \text{ or } \frac{4\sqrt{2}}{7}$$
$$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{18}{8\sqrt{2}} = \frac{9}{4\sqrt{2}} \text{ or } \frac{9\sqrt{2}}{8}$$
$$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{18}{14} = \frac{9}{7}$$
$$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{14}{8\sqrt{2}} = \frac{7}{4\sqrt{2}} \text{ or } \frac{7\sqrt{2}}{8}$$

3. 

SOLUTION:

The length of the side opposite θ is 9, the length of the side adjacent to θ is 4, and the length of the hypotenuse is $\sqrt{97}$.

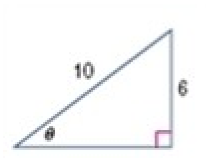
$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{9}{\sqrt{97}} \text{ or } \frac{9\sqrt{97}}{97}$$
$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{4}{\sqrt{97}} \text{ or } \frac{4\sqrt{97}}{97}$$
$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{9}{4}$$
$$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{\sqrt{97}}{9}$$
$$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{\sqrt{97}}{4}$$
$$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{4}{9}$$

5. 

SOLUTION:

The length of the side opposite θ is $\sqrt{165}$, the length of the side adjacent to θ is 26, and the length of the hypotenuse is 29.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{\sqrt{165}}{29}$$
$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{26}{29}$$
$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{\sqrt{165}}{26}$$
$$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{29}{\sqrt{165}} \text{ or } \frac{29\sqrt{165}}{165}$$
$$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{29}{26}$$
$$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{26}{\sqrt{165}} \text{ or } \frac{26\sqrt{165}}{165}$$

7. 

SOLUTION:

The length of the side opposite θ is 6 and the length of the hypotenuse is 10. By the Pythagorean Theorem, the length of the side adjacent to θ is 8.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{6}{10} \text{ or } \frac{3}{5}$$
$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{8}{10} \text{ or } \frac{4}{5}$$
$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{6}{8} \text{ or } \frac{3}{4}$$
$$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{10}{6} \text{ or } \frac{5}{3}$$
$$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{10}{8} \text{ or } \frac{5}{4}$$
$$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{8}{6} \text{ or } \frac{4}{3}$$

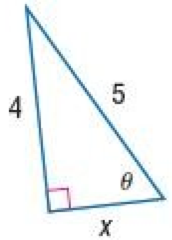
4-1 Right Triangle Trigonometry

Use the given trigonometric function value of the acute angle θ to find the exact values of the five remaining trigonometric function values of θ.

9. $\sin\theta = \frac{4}{5}$

SOLUTION:

Draw a right triangle and label one acute angle θ. Because $\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{4}{5}$, label the opposite side 4 and the hypotenuse 5.



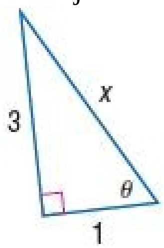
By the Pythagorean Theorem, the length of the side adjacent to θ is 3.

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{3}{5}$$
$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{4}{3}$$
$$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{5}{4}$$
$$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{5}{3}$$
$$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{3}{4}$$

11. $\tan \theta = 3$

SOLUTION:

Draw a right triangle and label one acute angle θ. Because $\tan \theta = \frac{\text{opp}}{\text{adj}} = 3 = \frac{3}{1}$, label the side opposite θ 3 and the adjacent side 1.



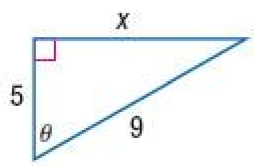
By the Pythagorean Theorem, the length of the hypotenuse is $\sqrt{3^2 + 1^2} = \sqrt{10}$.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{3}{\sqrt{10}} \text{ or } \frac{3\sqrt{10}}{10}$$
$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{1}{\sqrt{10}} \text{ or } \frac{\sqrt{10}}{10}$$
$$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{\sqrt{10}}{3}$$
$$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{\sqrt{10}}{1} \text{ or } \sqrt{10}$$
$$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{1}{3}$$

13. $\cos\theta = \frac{5}{9}$

SOLUTION:

Draw a right triangle and label one acute angle θ. Because $\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{5}{9}$, label the adjacent side 5 and the hypotenuse 9.



By the Pythagorean Theorem, the length of the side opposite θ is $\sqrt{9^2 - 5^2} = 2\sqrt{14}$.

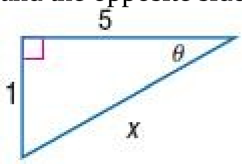
$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{2\sqrt{14}}{9}$$
$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{2\sqrt{14}}{5}$$
$$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{9}{2\sqrt{14}} \text{ or } \frac{9\sqrt{14}}{28}$$
$$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{9}{5}$$
$$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{5}{2\sqrt{14}} \text{ or } \frac{5\sqrt{14}}{28}$$

4-1 Right Triangle Trigonometry

15. $\cot\theta = 5$

SOLUTION:

Draw a right triangle and label one acute angle θ. Because $\cot\theta = \frac{\text{adj}}{\text{opp}} = 5 = \frac{5}{1}$, label the side adjacent to θ as 5 and the opposite side 1.



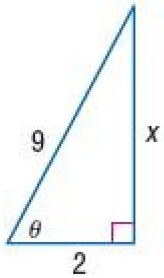
By the Pythagorean Theorem, the length of the hypotenuse is $\sqrt{5^2+1^2} = \sqrt{26}$.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{1}{\sqrt{26}} \text{ or } \frac{\sqrt{26}}{26}$$
$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{5}{\sqrt{26}} \text{ or } \frac{5\sqrt{26}}{26}$$
$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{1}{5}$$
$$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{\sqrt{26}}{1} \text{ or } \sqrt{26}$$
$$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{\sqrt{26}}{5}$$

17. $\sec\theta = \frac{9}{2}$

SOLUTION:

Draw a right triangle and label one acute angle θ. Because $\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{9}{2}$, label the hypotenuse 9 and the side opposite θ as 2.



By the Pythagorean Theorem, the length of the opposite side is $\sqrt{9^2 - 2^2} = \sqrt{77}$.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{\sqrt{77}}{9}$$
$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{2}{9}$$
$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{\sqrt{77}}{2}$$
$$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{9}{\sqrt{77}} \text{ or } \frac{9\sqrt{77}}{77}$$
$$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{2}{\sqrt{77}} \text{ or } \frac{2\sqrt{77}}{77}$$

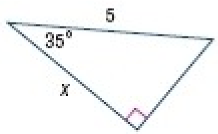
Find the value of *x*. Round to the nearest tenth, if necessary.

19. 

SOLUTION:

An acute angle measure and the length of the hypotenuse are given, so the sine function can be used to find the length of the side opposite the angle.

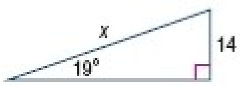
$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$
$$\sin 17° = \frac{x}{11}$$
$$11\sin 17° = x$$
$$3.2 \approx x$$

21. 

SOLUTION:

An acute angle measure and the length of the hypotenuse are given, so the cosine function can be used to find the length of the side adjacent to the angle.

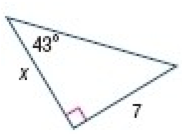
$$\begin{aligned}\cos\theta &= \frac{\text{adj}}{\text{hyp}}\\ \cos 35° &= \frac{x}{5}\\ 5\cos 35° &= x\\ 4.1 &\approx x\end{aligned}$$

23. 

SOLUTION:

An acute angle measure and the length of the side opposite it are given, so the sine function can be used to find the length of the hypotenuse.

$$\begin{aligned}\sin\theta &= \frac{\text{opp}}{\text{hyp}}\\ \sin 19° &= \frac{14}{x}\\ x\sin 19° &= 14\\ x &= \frac{14}{\sin 19°}\\ x &\approx 43.0\end{aligned}$$

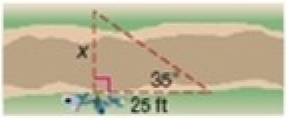
25. 

SOLUTION:

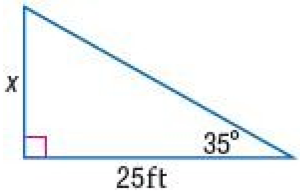
An acute angle measure and the length of the side opposite it are given, so the tangent function can be used to find the length of the side adjacent to the angle .

$$\begin{aligned}\tan\theta &= \frac{\text{opp}}{\text{adj}}\\ \tan 43° &= \frac{7}{x}\\ x\tan 43° &= 7\\ x &= \frac{7}{\tan 43°}\\ x &\approx 7.5\end{aligned}$$

27. **MOUNTAIN CLIMBING** A team of climbers must determine the width of a ravine in order to set up equipment to cross it. If the climbers walk 25 feet along the ravine from their crossing point, and sight the crossing point on the far side of the ravine to be at a 35º angle, how wide is the ravine?



SOLUTION:



An acute angle measure and the adjacent side length are given, so the tangent function can be used to find the length of the opposite side.

$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$
$$\tan 35° = \frac{x}{25}$$
$$25\tan 35° = x$$
$$17.5 \approx x$$

Therefore, the ravine is about 17.5 feet wide.

29. **DETOUR** Traffic is detoured from Elwood Ave., left 0.8 mile on Maple St., and then right on Oak St., which intersects Elwood Ave. at a 32° angle.

a. Draw a diagram to represent the situation.

b. Determine the length of Elwood Ave. that is detoured.

SOLUTION:

a. Draw a right triangle with acute angle 32º and opposite side length 0.8 mile. Label the side opposite the 32º angle Maple St., the hypotenuse Oak St., and the adjacent side Elwood Ave.

b. Because an acute angle and the length of opposite side are given, the tangent function can be used to find the adjacent side length.

$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$
$$\tan 32° = \frac{0.8}{x}$$
$$x\tan 32° = 0.8$$
$$x = \frac{0.8}{\tan 32°}$$
$$x \approx 1.28$$

Therefore, the length of Elwood Ave. that is detoured is about 1.3 miles.